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INSTRUMENT TRACKING FOR OCEAN ACOUSTIC TOMOGRAPHY EXPERIMENTS (U)

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TECHNICAL NOTE

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ABSTRACT (U)

Ocean acoustic tomography is a method for measuring the sound speed structure of a volume of ocean indirectly using the transmission of sound between instruments deployed within the area of interest. Since acoustic tomography is based on the measurement of travel time of transmitted signals, the data are very sensitive to the relative displacement of the instruments. This paper describes a navigation system which continuously tracks ocean acoustic tomography instruments. By this system, the contaminating effect of mooring motion on the measurement of travel times of acoustic signals is removed.

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Introduction

Ocean acoustic tomography is a method for measuring the sound speed structure of a volume of ocean indirectly using the transmission of sound between instruments deployed within the area of interest. The method can provide useful space and time resolution, and, it requires less ship time than that presently required to deploy and recover conventional oceanographic instruments. Acoustic tomography takes advantage of the facts that low frequency sound can travel long distances in the ocean, and, that travel time is a function of temperature and current velocity. Travel times of acoustic pulses can be interpreted, using inverse theory, to provide information about the oceanic circulation complete with its random motions.

Since acoustic tomography is based on the measurement of travel times of transmitted signals, the data are very sensitive to the relative displacement of the instruments. Given a typical sound speed of 1500 m/sec, 15 m of error in the estimation of the length of an acoustic ray adds 10 msec of travel time error. This is important when compared with 40 msec, the expected order of travel time changes due to the intermediate, or oceanic mesoscale field. In other words, mooring motion can introduce a variability about the travel times which can swamp the mesoscale variations. The structure of a typical acoustic mooring is shown in Fig.1.

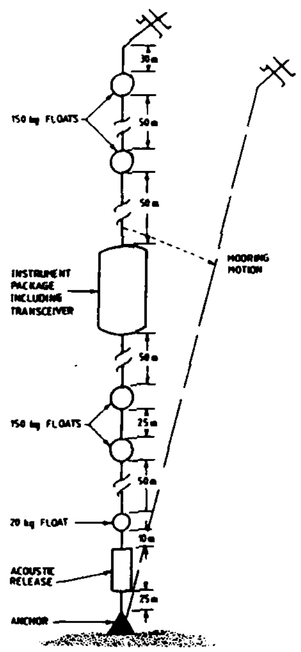

Fig.1 Structure of an acoustic mooring at rest depth. Note that a horisontal shift, as a result of mooring motion, is accompanied by a deepening of the instrument.

The mooring motion is monitored by means of an acoustic navigation system. This system consists of a micro-processor controlled transceiver which is mounted as part of the sensor package, and three recoverable transponders anchored on the bottom, approximately 2000 m from the mooring anchor. At pre-determined intervals the transceiver simultaneously interrogates the three transponders and measures the round-trip travel times. These data are recorded along with the time and date so that after processing, the path of the mooring may be reconstructed. A scheme of the geometry is shown in Fig.2. A similar system has been described previously by Nowak and Mealy.(1)

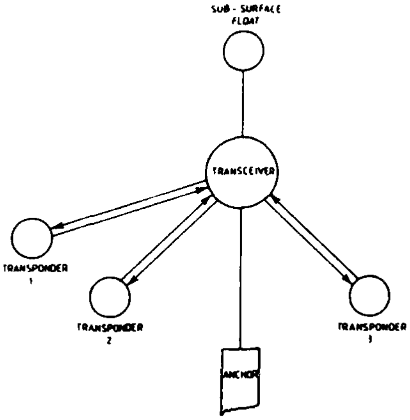

Fig.2 Mooring navigation scheme

Inferences in the oceanographic field by acoustic tomography techniques are questionable unless the recorded arrival times, which are used in tomography to infer oceanic structure, are known with respect to the position of the instrument. The arrival times between arrays of sources and receivers are separate and distinct from the arrival times between the three transponders and the transceiver which are used to track the motion of a mooring. It is the latter type of arrival times which are the subject of this report.

This technical note reports on the work which was done as a contribution to a French/US ocean acoustic tomography experiment which was conducted in the Gulf Stream by scientists from IFREMER/WHOI in 1987 (the results of which are yet to be published). This note details an analytical expression which may be used, for speed and simplicity, as a close approximation to the exact or ray trace solution for the purpose of monitoring mooring motion.

Method

Once the path of a ray has been traced from the transceiver to a transponder it is possible to calculate the travel time, by integrating along the ray path. A sound speed profile, which is representative of the Gulf Stream, is shown in Fig.3. This profile was input to a ray trace program to find arrival times for horizontal ranges between 500 m and 4500 m.

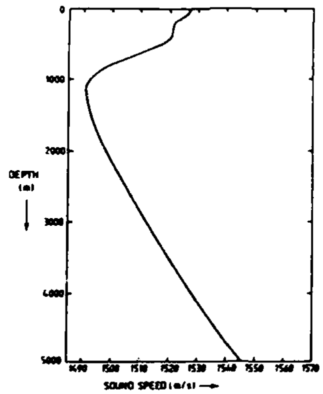

Fig.3 Typical Gulf Stream sound speed profile

Arrival time (τ) may be expressed in terms of its dependent variables, the depth of the transceiver (z_S) and the horizontal range between the transceiver and the transponder (r), thus $\tau(r, z_S)$. The ray trace program was run several times for selected source depths, ranging from 1000 m to 2000 m. Here depth is assumed to increase downwards with sero depth occurring at the surface. The arrival times obtained are said to represent the exact solutions. It is possible to construct a curve which relates arrival time to horizontal range for a particular source depth. Examples of three such curves are drawn in Fig.4.

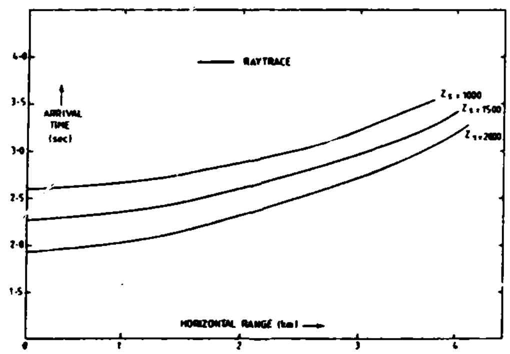

Fig.4 Arrival time is plotted against horizontal range for various source depths.

Obviously, an infinite number of range solutions is possible for any arrival time if depth information is unavailable. However, if three arrival times are known for a single given source depth, it is possible to interpolate to determine horisontal range. A faster method of determining horizontal range, given arrival time and source depth, is to derive an analytical expression which approximates the exact solution.

The methodology used for determining horisontal range was to make a comparison between approximations obtained from three different analytical expressions with results obtained from the ray trace solution. These expressions for arrival time, approximate the sound speed profile, and or, the exact ray path. It is convenient in this section of the report to refer to Table 1, in which each expression is placed according to these approximations. Expressions 1 and 2 were found to be unsatisfactory approximations to the exact solution for the purposes of mooring motion. Their derivation is given in Annexes 1 and 2 respectively, for completeness and for use in other possible applications. All parameters shown in expressions 1 and 2 are similarily defined in Annexes 1 and 2.

Expression 3 proved to be a close approximation to the exact solution and its derivation and validation follows.

Table 1 Analytical expressions which were tested as approximations to the ray trace solution

Sound Speed Profile / Ray Path	Approximation	Exact Solution
Approximation	1 $\tau = \frac{1}{b\cos\alpha} \ln \frac{C_o}{C_o - bh}$	3 $\tau = T_v \sqrt{1 + (\frac{r}{h})^2}$
Exact Solution	2 $\tau = \frac{1}{b} \ln \frac{(w_f(+\cos\alpha_i)}{(w_i(1+\cos\alpha_j)}$	4 *Ray Trace*

Approximation to Ray Trace Solution

Consider a ray travelling from S to R through the ocean volume in which the speed of sound C, varies with depth, z. The ray leaves the source at a depth, z_S and an angle, α_i, where α_i is the initial angle as measured with respect to the vertical. The relevant geometry is given in Fig.5.

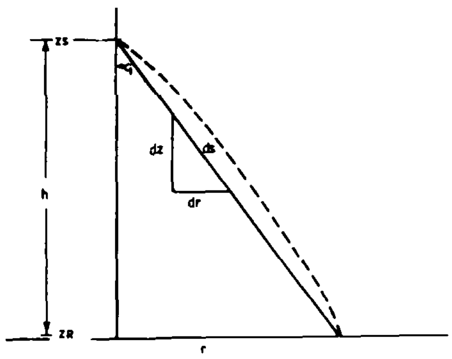

Fig.5 Sketch of how a ray trajectory may be approximated by a straight ray path

The integral travel time, between the source and the receiver in the vertical direction, (T_v) is found by integrating the sound speed profile from the source depth to the receiver depth. Assuming a straight ray path, the ray reaches R, the depth of the transponder, after an integral travel time which is given by:

$$T_v = \int_{z_S}^{z_R} \frac{dz}{C(z)}$$

The travel time is described by

$$\tau = \int_{z_R} \frac{ds}{C(z)}$$

where ds is the distance measured along the ray, and the integral is taken along a straight ray path.

Since

$$ds = \frac{dz}{\cos(\alpha_i)}$$

the arrival time may be written with respect to the distance along the ray SR

$$\tau = \frac{1}{\cos(\alpha_i)} \int_{z_S}^{z_R} \frac{dz}{C(z)}$$

Now substituting for T_v

$$\tau = \frac{T_v}{\cos(\alpha_i)}$$

$$\frac{1}{\cos(\alpha_i)} = \sqrt{1 + \tan(\alpha_i)}$$

$$= \sqrt{1 + (\tfrac{r}{h})^2}$$

where $r = h\tan(\alpha_i)$

and $h = z_S - z_R$

thus $\tau = T_v \sqrt{1 + (\tfrac{r}{h})^2}$

Simpson's composite algorithm is called to approximate the integral travel time, T_v using a number of sub-intervals, m. Annex 3 gives details of this numerical integration technique and values of T_v for source depths between 1000 m and 3000 m. The water depth is assumed to be 5000 m. Fig 6 presents the results graphically.

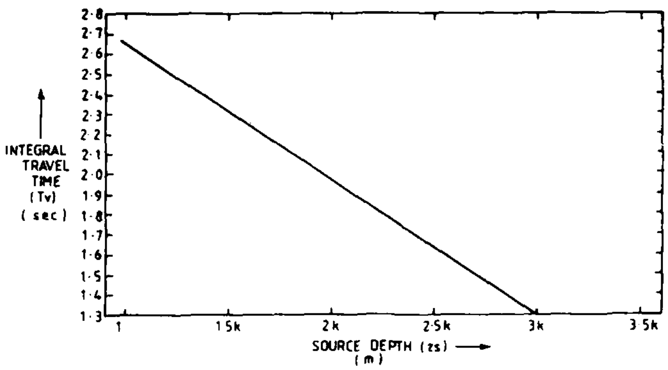

Fig6 Graph showing the near linear relation between the integral travel time, T_v and source depth, z_S.

The analytical expression $\tau = T_v\sqrt{1 + (\frac{r}{h})^2}$ closely approximates the exact solution for source depths between 1000 m and 3000 m and horizontal ranges between 1000 m and 5000 m. A comparison was made between arrival times obtained with this expression and those obtained by using the ray trace program. The largest difference in arrival times between the two solutions is found at 5000 m range (0.6 msec). This represents a difference in range of approximately 0.9 m. In all cases the differences in travel time are small. It is predictable that as the ranges are extended further, a straight ray trajectory will become an increasingly worse approximation to the exact solution.

The mooring tracking system is able to tolerate the size of the differences observed, which are in the order of tenths of milliseconds. In fact, the system is more likely to be limited by propagation loss out to 5000 m, than by the use of an approximation to the ray trace solution.

Arrival Time Expressed in Terms of Instrument Position

Once the arrival time has been closely approximated by an analytical expression, it only remains necessary to introduce the other information which is available to the navigation system, that is, the known positions of the transponders to arrive at a solution for the mooring position. Although it is impossible to assign positions "a priori" to transponders, once deployed,the positions of the transponders(x_i, y_i, z_i) are determined by survey. For our purposes we will assume that the anchor is at the origin. Additional assumptions are that the ocean bottom is flat and 5000 m below the surface.

The arrival times are input into an iterative program which gives the position of the transceiver in terms of its cartesian coordinates. The program solves for x,y and z by using the following equation where $i = 1, 2, 3$.

$$T_v = \frac{\tau_i}{\sqrt{1 + \frac{(x-x_i)^2 + (y-y_i)^2}{(z_i - z)}}}$$

The iterative routine finds a zero of a system of three functions in x, y, z by a modification of the Powell hybrid method(2). This routine is further explained in Annex 4. which provides a numerical solution for three non-linear equations relating arrival time to instrument position. T_v is calculated for any depth between 1200 m and 2000 m by means of a cubic spline interpolation. A listing of the output from the cubic spline interpolation is given in Annex 5. The FORTRAN source code for the mooring tracking program is given in Annex 6 and a summary of the inputs and the processing steps which are required to achieve a solution is given in Fig7. Numerical Algorithms Group(NAG) maths library routines(3) are called to perform both the iteration and the interpolation.

Curve Fitting for the Integral Travel Time Term

The problem of fitting a curve for the integral travel time term is solved by approximating the values for the integral travel time between transceiver and transponder to a function, $f(x)$. In addition, it would be advantageous if the navigation system was able to make a close approximation of the instrument position if depth information was not known. A close approximation was made by fitting the travel time data to a least squares cubic spline curve. A least squares cubic spline is used in preference to a least squares polynomial fit since there is only one independent variable.

Under normal routine operation of the mooring tracking system, depth information is available from the pressure sensor which is mounted as part of the instrument package and the navigation system will call the cubic spline routine to evaluate T_v. However, if the pressure sensor fails so that depth information is lost, a back-up system is available to replace the spline interpolation.

In this case, since the required function is almost linear and extremely well-behaved, one can adopt an expression of the form $A + Bz_s$ in order to determine T_v. A single linear equation is valid for a distance of approximately 200 m. For example, assume that a mooring which has a rest depth of 1500 m does not move deeper than 1700 m, then T_v may be approximated to a linear fit for $1700 > z_S > 1500$ as $T_v \simeq 3.31010 - 6.69 \times 10^{-4} z_S$. Further examples of linear fits between certain possible intervals are given in Table 2.

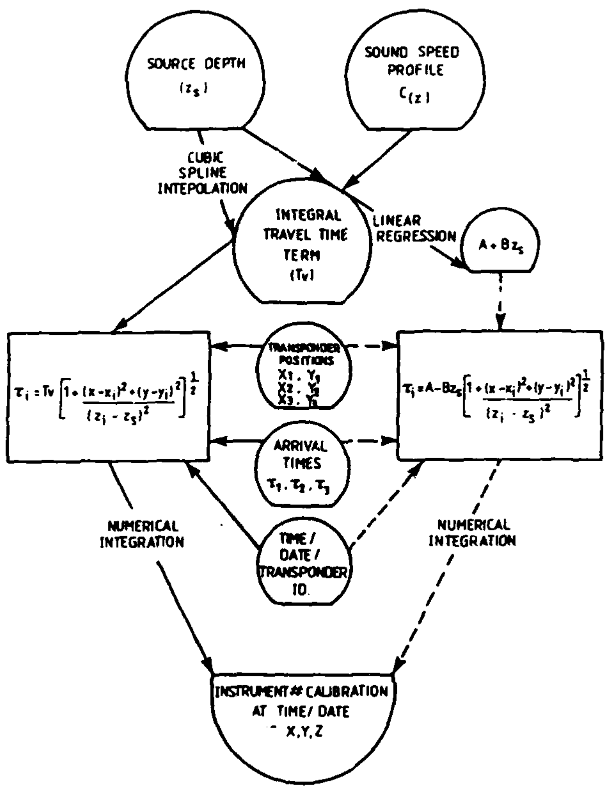

Fig7 Scheme of the inputs and processing required to arrive at a solution for transceiver position. Solid lines represent main path for which source depth is accurately known. Stippled lines indicate back-up path for which source depth is approximately known.

Rest Depth (m)	A	B
1200	3.3136	-6.705×10^{-4}
1300	3.3402	-6.895×10^{-4}
1400	3.3109	-6.695×10^{-4}
1500	3.3101	-6.690×10^{-4}
1600	3.3101	-6.690×10^{-4}
1700	3.3092	-6.685×10^{-4}
1800	3.3074	-6.675×10^{-4}

Table 2. Coefficients obtained by fitting the integral travel time term, T_v to the source depth, z_S, linearly

The advantage of fittin· T_v linearly, in the absence of depth information, is that the iteration routine is able to solve for z_S as a third unknown, in each of the three equations. A close estimate of the instrument position is possible because the coefficients A and B change slowly with depth because the sound speed profile is close to linear at these depths.

To reiterate, with normal navigation system operation, that is, complete with instrument depth information, curve fitting for T_v against z_S is best done with a least squares cubic spline routine. If depth information is known only to within 200 m accuracy it is necessary to use a routine which relies on a close linear fit for T_v versus z_S.

Results

A simulated survey problem with transponders at B_1 (1000, 800, 0) B_2 (1100, −900, 0) and B_3 (−1200, −200, 0) was designed to test the mooring tracking system. Fig. 8 shows the instrument in its unperturbed position at M (0, 0, 1200). The ray trace solution was used to derive arrival times between each of the 3 transponders and the transceiver. Accurate positioning of the instrument was obtained when the instrument was without any mooring motion, as shown by $x(-0.055), y(0.055), z(1199.34)$

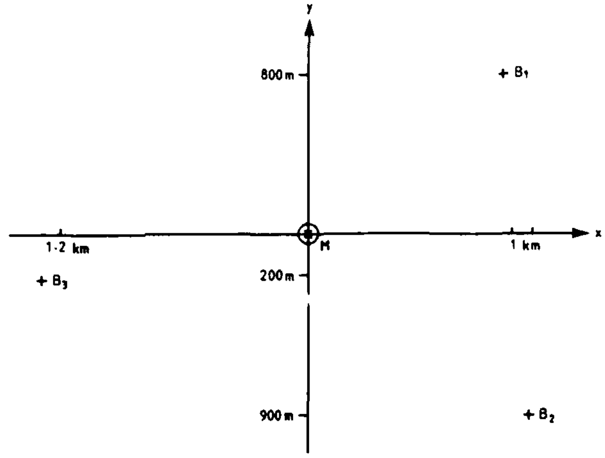

Fig 8. Transponder positions with respect to the mooring anchor.

The moorings were then perturbed with respect to lean angle and lean direction. For both maximum generality and simplicity the moorings were assumed to be rigid and the geometry follows the lead given by Cornuelle (4). Fig. 9 shows the instrument position changes as a result of mooring lean.

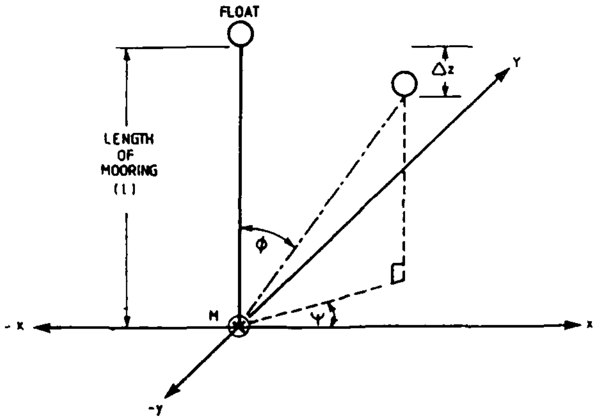

Fig. 9. Mooring lean geometry $\Delta x = l \sin\phi \cos\psi$, $\Delta y = l \sin\phi \sin\psi$, $\Delta z = \frac{l \sin^2\phi}{2}$. Adapted from Cornuelle(4)

Fig. 10 presents the results of this simulation. Ranges are from 400 m to 2000 m. The ray tracing for this work was done by specifying the starting point and initial direction of the ray and treating it as an initial value problem. With this particular ray tracing program there was no control over the point of emergence of any particular ray. To overcome this problem, the position of the receivers was adjusted to ensure that an arrival time was determined for all lean angles and directions. Arrival times and ranges were evaluated to milliseconds and centimetres respectively. The system accuracy suffered if the measurements were not made to this precision. The extremely small percentage errors show that the tracking system was able to accurately keep track of the mooring motion even when the mooring was subjected to significant lean.

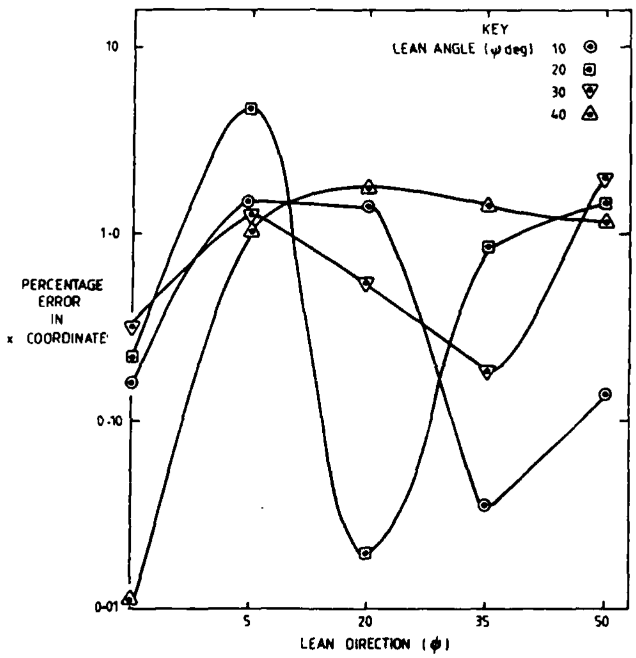

Fig 10a Results of simulation, percentage error versus lean direction for x coordinate.

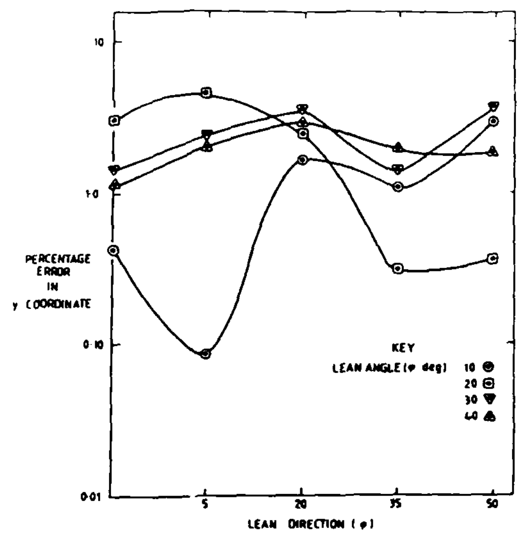

Fig. 10b Results of simulation, percentage error versus lean direction for y coordinate.

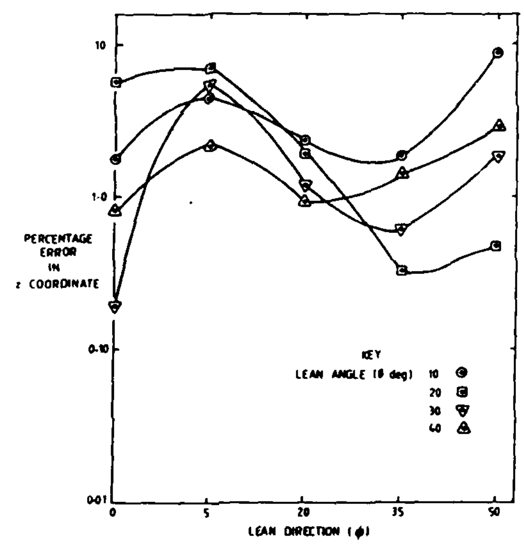

Fig. 10c Results of simulation, percentage error versus lean direction for z coordinate.

Acknowledgements

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Annex 1: An analytical expression which approximates the ray trace solution by assuming approximations to both the sound speed profile and the ray trajectory.

Expression 1 of Table 1 takes an approximation for the sound speed profile with a straight line approximation for the ray path. To obtain an expression for arrival time, an incremental step in the ray trajectory, ds, is taken, $ds = \frac{dz}{\cos(\alpha_i)}$

A linear approximation for the sound speed gradient may be written in the form

$C_z = C_o + bz$

where C_z is the sound velocity as a function of depth, b is the sound speed gradient, and C_o is a constant.

Arrival time, $\tau = \frac{1}{\cos(\alpha_i)} \int_0^h \frac{dz}{C_o - bz}$

where h is the distance between the transceiver and the transponder.

Thus $\tau = \frac{1}{b\cos(\alpha_i)} \frac{\ln(C_o)}{C_o - bh}$

$bh = C_o(1 - e^{-b\tau\cos(\alpha_i)})$

horizontal range, r, is thus

$r = \frac{C_0}{b} \tan(\alpha_i)\ 1 - e^{-b\tau\cos(\alpha_i)}$

The required inputs for this expression are an initial launch angle and arrival time, the expression calculates the horizontal shift of the transceiver for a given depth. Fig A1 shows the differences in range estimates between an expression which approximates both the sound speed profile and the ray path, and the exact solution. Two source depths are drawn. Range differences are large for all initial launch angles. The expression is therefore unsuitable to use for the purposes of mooring motion.

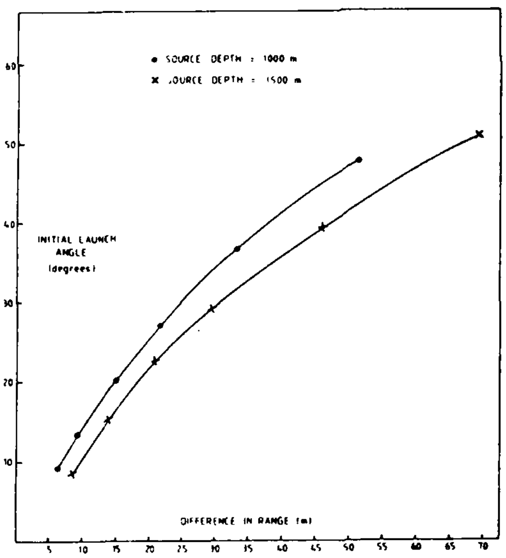

Fig A1 Launch angle versus range differences

Annex 2: An analytical expression which approximates the ray trace solution by assuming an approximation to the sound speed profile.

Expression 2 of Table 1 is a ray trace approximation which uses an exact ray path and a linear approximation for the sound speed profile. The sound speed approximation takes the form

$$C_z = C_o + bz$$

where, C_o is a constant and b is the sound speed gradient

$$b = \frac{d[C(z)]}{dz}.$$

C_z is the speed of sound at depth z. The expression for arrival time is taken from Clay and Medwin (5). Fig A2.1 shows the circular ray path for a linear dependence of sound speed on depth.

$$\tau = \frac{1}{b} \ln \left[\frac{w_f (1 + \cos \alpha_i)}{w_i (1 + \cos \alpha_f)} \right]$$

where

$$w_f = (z_R - z_S) + \frac{C_s}{b}$$

and

$$w_i = \frac{C_s}{b}$$

α_i is the initial angle as measured with the vertical axis and α_f is the final angle, also measured with respect to the vertical.

z_R is the transponder depth and z_S is the transceiver depth.

The expression requires a range and arbitrary α_i as input, to determine arrival time. In order to find eigenrays for the derived arrival time, the angles α_i and α_f are put into a second expression which gives the horizontal range, r, assuming the true ray trajectory.

$$r = \frac{1}{ab} [\cos(\alpha_i) - \cos(\alpha_f)]$$

where

$$a = \frac{\sin(\alpha_i)}{C_i}$$

and

C_i is the speed of sound at the transceiver depth.

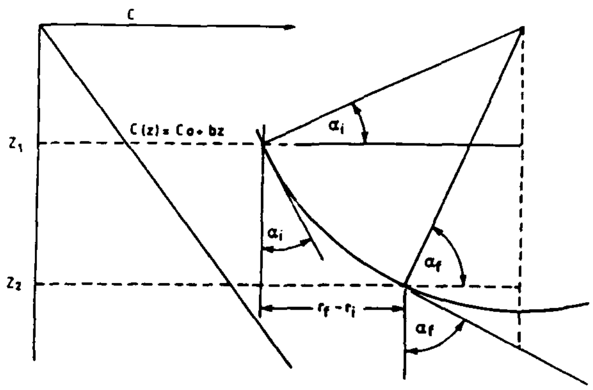

Fig A2.1 Circular ray path for a linear dependence of sound speed on depth.

The expression may then be used to construct arrival times versus range curves for various transceiver depths. However, the range scales are such that it is difficult to obtain sufficient resolution to determine the difference in range between the approximation and the trace from these curves. The difference in range is best shown if a direct comparison is made between ranges and arrival times obtained with the expression with those obtained after interpolation of the ray trace solution. Table A2 presents examples of this comparison.

Source Depth = 1500 m

Speed of Sound = 1493.1 m/s

Analytical Expression	Ray Trace	Range Difference (m)
2047.828 m 2.669 s	2037.5 m 2.669 s	10.33
3629.403 m 3.318 s	3618.75 m 3.318 s	10.66
4172.547 m 3.584 s	4162.5 m 3.584 s	10.05
4726.363 m 3.870 s	4718.75 m 3.870 s	7.61

Source Depth = 1000 m

Speed of Sound = 1492.8 m/s

Analytical Expression	Ray Trace	Range Difference (m)
712.954 m 2.674 s	625.00 m 2.674 s	87.95
1530.816 m 2.819 s	1487.50 m 2.819 s	43.32
3085.750 m 3.325 s	3056.25 m 3.325 s	29.50
4691.562 m 4.057 s	4662.50 m 4.057 s	29.06

As expected, the difference in range is greater for shallower transceiver depths. Clearly, the differences in horizontal range between the approximation and the exact solution are too large for this method to be considered as a possible option for use in tracking mooring motion.

Annex 3: A numerical integration technique used to evaluate the integral travel time term, T_v.

Several numerical methods are available for integrating the sound speed profile between z_S and z_R; the method which is used here is Simpson's composite algorithm. The procedure is to divide the interval into n sub-intervals and use Simpson's rule on each pair of consecutive sub-intervals. Since each application of Simpson's rule requires 2 intervals, n must be an even integer, that is, $n = 2m$ for some integer m.

With

$$h = \frac{z_S - z_R}{2m}$$

and

$$z_S = x_0 < \ldots < x_{2m} = z_R$$

where

$$x_j = x_0 + jh \quad \text{for each } j = 0, 1, \ldots, 2m$$

$$\int_{z_R}^{z_S} f(x)dx = \sum_{j=1}^{m} \int_{x_{2j-2}}^{x_{2j}} f(x)dx$$

$$\int_{z_R}^{z_S} f(x)dx = \sum_{j=1}^{m} \left[\frac{h}{3} f(x_{2j-2}) + 4f(x_{2j-1}) + f(x_{2j})\right] - \delta$$

where δ is an error term.

Table A3 gives values for the integral travel time between transceiver and transponder assuming a straight ray path and a water depth of 5000 m.

Table A3

Source Depth (z_S)	No. of sub-intervals (m)	Integral travel time (T_v)
1000	200	2.6417
1200	190	2.5077
1400	180	2.3736
1600	170	2.2397
1800	160	2.1059
2000	150	1.9724
2200	140	1.8390
2400	130	1.7059
2600	120	1.5731
2800	110	1.4405
3000	100	1.3081

Annex 4: A numerical solution for three non-linear equations relating arrival time to instrument position.

From the analytical expression which closely approximates the ray trace solution

$$\tau_i = T_v \sqrt{1 + \frac{(r_i)^2}{z_R - z_S}}$$

and distance formula

$$r_i{}^2 = (x - x_i)^2 + (y - y_i)^2$$

substitute for arrival time

$$\tau_i = T_v \sqrt{1 + \frac{(x - x_i)^2 + (y - y_i)^2}{(z_R - z_S)^2}}$$

for $i = 1, 2, 3$

So, for instrument position X, Y, Z

$$X = (x - x_2)^2 - (z_R - z_S)^2 (\tfrac{\tau_2}{T_v})^2 + (z_R - z_S)^2 + (y - y_2)^2$$

$$Y = (y - y_3)^2 - (z_R - z_S)^2 (\tfrac{\tau_3}{T_v})^2 + (z_R - z_S)^2 + (x - x_3)^2$$

$$Z = T_v \sqrt{1 + \frac{(x - x_1)^2 + (y - y_1)^2}{(z_R - z_S)^2}} - \tau_1$$

An iterative routine finds a zero of a system of 3 equations in 3 variables by a modification of the Powell hybrid method. It requires as input, initial estimates for X and Y which are within approximately 500 m of the true instrument position.

Since the term T_v is depth dependent, a close estimate for z_S is essential. This routine chooses the correction at each step as a convex combination of the Newton and scaled gradient directions. Under reasonable conditions, this guarantees global convergence for starting points far from the solution and a fast rate of convergence. The Jacobian is updated by the rank-1 method of Broyden. At the starting point the Jacobian is approximated by forward differences, but these are not used again unless the rank-1 method fails to produce satisfactory progress

Annex 5: Cubic spline fit to arbitrary data.

Input Data

Number of data points = 31   Number of intervals = 4

Unit weighting factors

R	ABSCISSA X(R)	ORDINATE Y(R)
1	1200	2.5077
2	1220	2.4943
3	1240	2.4809
4	1260	2.4674
5	1280	2.4540
6	1300	2.4406
7	1320	2.4272
8	1340	2.4138
9	1360	2.4004
10	1380	2.3870
11	1400	2.3736
12	1420	2.3602
12	1440	2.3468
13	1460	2.3334
14	1480	2.3200
15	1500	2.3066
16	1520	2.2932
17	1540	2.2798
18	1560	2.2664
19	1580	2.2530
20	1600	2.2397
21	1620	2.2263
22	1640	2.2129
23	1660	2.1995
24	1680	2.1861
25	1700	2.1728
26	1720	2.1594
27	1740	2.1460
28	1760	2.1326
29	1780	2.1193
30	1780	2.1193
31	1800	2.1059

Results

J	KNOT K(J + 2)	B SPLINE COEFF C(J)
1	.	2.5077
2	1200	2.4853
3	1300	2.4183
4	1300	2.3066
5	1700	2.1950
6	1800	2.1282
7	.	2.1059

Cubic spline approximation and residuals

ABSCISSA	APPROXIMATION	RESIDUAL
1200	2.5077	.22489E−6
1220	2.4943	.45562E−6
1240	2.4809	.55100E−7
1260	2.4674	−.88447E−7
1280	2.4540	.26823E−6
1300	2.4406	.27963E−6
1320	2.4272	.90167E−7
1340	2.4138	−.19607E−6
1360	2.4004	−.48508E−6
1380	2.3870	.31715E−6
1400	2.3736	−.42864E−6
1420	2.3602	−.42864E−6
1440	2.3468	.21135E−6
1460	2.3334	.31860E−6
1480	2.3200	−.12862E−7
1500	2.3066	−.68905E−6
1520	2.2932	−.62783E−6
1540	2.2798	.20535E−6
1560	2.2664	−.16684E−6
1580	2.2664	−.16684E−6
1580	2.2530	.27828E−6
1600	2.2397	.56338E−6
1620	2.2263	.71112E−6
1640	2.2129	.74419E−6
1660	2.1995	−.31475E−6
1680	2.1861	−.44302E−6
1700	2.1728	−.61795E−6
1720	2.1594	.73370E−6
1740	2.1460	−.35173E−6
1760	2.1326	.49676E−6
1780	2.1193	.19922E−5
1800	2.1059	−.10024E−6

Annex 6: A listing of the mooring tracking program. The source code is written in FORTRAN for an IBM PC.

```
C       Program TRACK.FOR
C
C
C       This program finds a zero of a system of 3 non-linear
C       functions in 3 variables by a modification of the Powell
C       hybrid method.
C
C       Subroutine SPLINE is called to find Tv for a given source
C       depth betwen 1200m and 2000m.
C
C       N - integer specifying the number of equations.
C       X - real array of DIM(N), it contains the point at which
C         the functions are to be evaluated. Before entry, X(j)
C         must be set to a guess at the jth component of the
C         solution ( j = 1,2,..,n). On exit j contains the
C         final estimate of the solution vector.
C         FVEC - real array of DIM(N), it must contain the
C           value of the ith fn. evaluated at the point X.
C         IFLAG - integer, if program termination is required
C           it is set negative.
C          TOL - real, XTOL must specify the accuracy in X to
C            which the solution is required, the
C            recommended value is the square root of the
C            machine precision.
C          WA - real array of DIM(LWA), used as workspace.
C          LWA - integer, LWA = 1/2 N * (3 * N + 13)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        INTEGER J,N,NOUT
        COMMON /ITER/ T1,T2,T3,Tv,X1,X2,X3,Y1,Y2,Y3,R1
        DOUBLE PRECISION WA(33)
        CHARACTER*1 LF,CR
        DOUBLE PRECISION DSQRT,T1,T2,T3,Tv,X1,X2,X3,Y1,Y2,Y3
        DOUBLE PRECISION R1,FNORM,TOL,F05ABF,X02AAF,X(3),FVEC(3)
        EXTERNAL FCN
        DATA NOUT /16/
        OPEN (16,FILE = 'TRACK.RES', STATUS = 'NEW')
        WRITE(NOUT,999)
        LF = CHAR(10)
        CR = CHAR(13)
C
        WRITE(NOUT,999)
```

```
      WRITE(*,*)'INPUT THE 3 ARRIVAL TIMES (sec)', LF
      READ (*,*) T1,T2,T3
      WRITE(*,*)'INPUT THE OCEAN DEPTH (m)',LF
      READ(*,*) R1
      WRITE(*,*)'INPUT THE X AND Y COORDINATES OF THE;'
      WRITE(*,*)'3 TRANSPONDERS, ASSUME THE ANCHOR;'
      WRITE(*,*)'IS AT THE ORIGIN e.x. 1000,-1400;'
      WRITE(*,*)' -900,1100;'
      WRITE(*,*)' -1400,-1600',LF
      READ(*,*) X1,Y1,X2,Y2,X3,Y3
C
C     THE FOLLOWING STARTING VALUES GIVE A ROUGH SOLUTION
C
      WRITE(*,*)'INPUT THE LAST KNOWN X COORDINATE;'
      WRITE(*,*)'OF THE TRANSCEIVER (m)', LF
      WRITE(*,*)'INPUT THE LAST KNOWN Y COORDINATE;'
      WRITE(*,*)'OF THE TRANSCEIVER (m)', LF
      WRITE(*,*)'INPUT THE DEPTH OF THE TRANSCEIVER (m)', LF
      READ(*,*) X(3)
C
      CALL SPLINE(X(3),Y)
      Tv = Y
      N = 3
      TOL = DSQRT(X02AAF(0.00)
      IFAIL = 0
      LWA = 33
      CALL C05NBF(FCN,N,X,FVEC,TOL,WA,LWA,IFAIL)
      FNORM = F05ABF(FVEC,3)
      WRITE(NOUT,998) FNORM,IFAIL,(X(J).J = 1,3)
      WRITE (NOUT,996) Tv,R1,T1,T2,T3,X1,Y1,X2,Y2,X3,Y3
      STOP
999     FORMAT(4(1X/),36H TRACK-MOOR EXAMPLE PROGRAM RESULTS /1X)
998     FORMAT(5X,31H FINAL L2 NORM OF THE RESIDUALS,E12.4//5X,
     1 15H EXIT PARAMETER,I10//5X,27H FINAL APPROXIMATE SOLUTION//
     1 (5X, 3E12.6)
996     FORMAT(/6H Tv =,E20.5/6H R1 =,E20.5/6H T1 =,E20.5/6H
     1 T2 =,E20.5/6H T3 =,E20.5/9H X1,Y1 =,2E20.5/9H
     1 X2,Y2 =,2E20.5/9H X3,Y3 =,2E20.5/)
      END
C
      SUBROUTINE SPLINE(XARG,FIT)
C
C       This routine computes a weighted least squares
```

```
C      approximation to an arbitrary set of data points
C      by a cubic spline with knots prescribed by the user
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      INTEGER NOUT,M,NCAP,NCAP2,NCAP3,NCAP7,J
      INTEGER IFAIL,R,NIN
      DOUBLE PRECISION SS,XARG,FIT,X(41),Y(41),W(41)
      DOUBLE PRECISION C(41),WORK(41),WORK2(4,41),K(41)
      DATA NOUT /16/,NIN /25/
      OPEN(25,FILE = 'SPLINE2.INP',STATUS = 'OLD')
      IFAIL = 0
C      M is the number of data points
C      NCAP is the number of pre-defined intervals
C      The knots are in the array K
C
      READ(NIN,99997) NCAP
      NCAP2 = NCAP + 2
      NCAP3 = NCAP + 3
      NCAP7 = NCAP + 7
      IF (NCAP .EQ. 1) GO TO 40
      READ(NIN,99995) X(R),Y(R)
      W(R) = 1.0
80     CONTINUE
      IFAIL = 1
      CALL E02BBF(NCAP7,K,C,XARG,FIT,IFAIL)
      IF (IFAIL .NE. 0) GO TO 300
      GO TO 320
300     WRITE (NOUT,99979) R,XARG
320     CONTINUE
RETURN
C
99997     FORMAT(I4)
99996     FORMAT(E21.11)
99995     FORMAT(2E21.11)
99979     FORMAT(1H,E20.5,23H ARGUMENT OUTSIDE RANGE)
      END
C
      SUBROUTINE FCN(N,X,FVEC,IFLAG)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      INTEGER IFLAG,N
      DOUBLE PRECISION FVEC(N),X(N)
      COMMON /ITER/ T1,T2,T3,Tv,X1,X2,X3,Y1,Y2,Y3,R1
C
```

```
      X(1) = X
      X(2) = Y
      X(3) = Z
      FVEC(1) = (Tv*DSQRT(1+((X(1)-X1)**2+(X(2)-Y1)
     1 **2)(R1-X(3))**2))-T1
      FVEC(2)=(X(1)-X2)**2-(R1-X(3))**2*((T2/Tv)**2)
     1 + (R1-X(3))**2+(X(2)-Y2)**2
      FVEC(3)=(X(2)-Y3)**2-(R1-X(3))**2*((T3/Tv)**2)
     1 + (R1-X(3))**2+(X(1)-X(3)**2
C
      RETURN
      END
```

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OCEAN ACOUSTIC TOMOGRAPHY SHORT-RANGE NAVIGATION MOORING MOTION	

16. Abstract

Ocean acoustic tomography is a method for measuring the sound speed structure of a volume of ocean indirectly using the transmission of sound between instruments deployed within the area of interest. Since acoustic tomography is based on the measurement of travel time of transmitted signals the data are very sensitive to the relative displacement of the instruments.

This paper describes a navigation system which continuously tracks ocean acoustic tomography instruments. By this system, the contaminating effect of mooring motion on the measurement of travel times of acoustic signals is removed.

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